Analysis of a portable Wattmeter

by Frederick Glenn, K9SO

Figure 1 shows a simplified diagram of a directional coupler. The addition of a diode detector would make this a directional detector. The capacitor is shown as dotted since it represents a parasitic capacitance from the pickup loop to the center conductor of the coaxial cable. The pickup loop itself may be as simple as a straight piece of wire and the terminating resistance is assumed to be non-inductive at the frequencies of interest. The pickup loop may be rotated 180^0 to be on the left hand side of the resistor-capacitor divider. Coupler analysis is often done in terms of the even and odd impedance values, but here a circuit analysis approach will be taken.

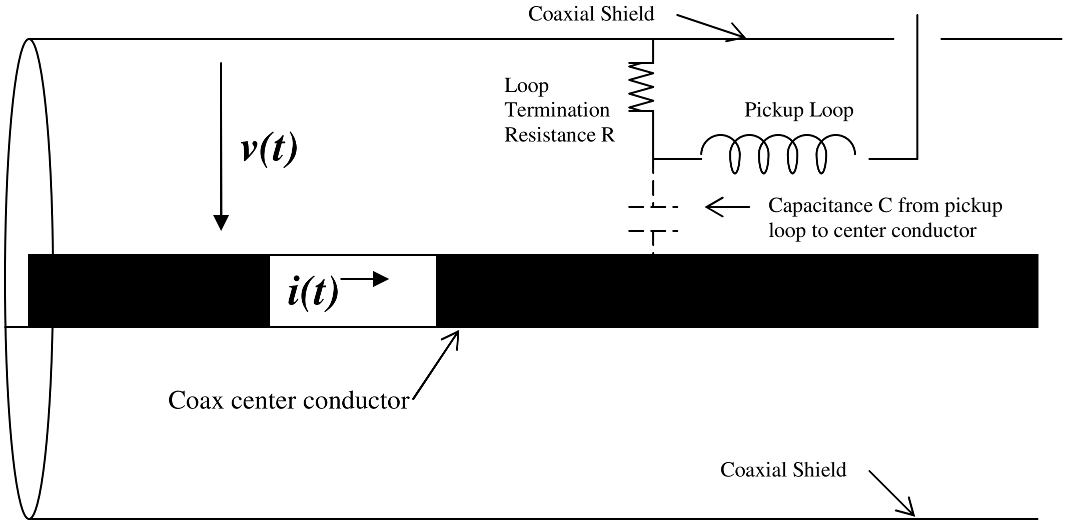

Figure 1: a simplified schematic of a directional coupler

Let ***v(t)*** and ***i(t)*** be the instantaneous and time-varying voltages and currents at a particular point of interest along the transmission line. Note that ***v(t)/i(t)*** will then define an impedance **Z** at this particular point of interest. In the special case of a transmission line terminated in its own characteristic impedance (e.g., no reflected wave), this become ***v(t)/i(t) = Z₀*** , the characteristic impedance of the line itself.

[In this paper, the time-varying "a function of (t)" designations will not be shown for simplicity and time-varying signals such as ***v(t)*** will simply be shown as lower case letters in italics: ***v***

Furthermore, ***v*** and ***i*** will refer to the voltages and currents of the transmission line. Other voltages (such as voltages across components) will be represented by ***e***. One must keep in mind that these are also time-varying voltages and could otherwise be written as ***e(t).***]

Figure 2 below defines some of these voltages and currents. Here, the inductance is represented by M, the mutual inductance between the center conductor and the pickup loop (it also has self inductance L) and $X_c = 1/j\omega C$. The physical length is assumed to be small compared with the wavelength and, of course, $\omega = 2\pi f$.

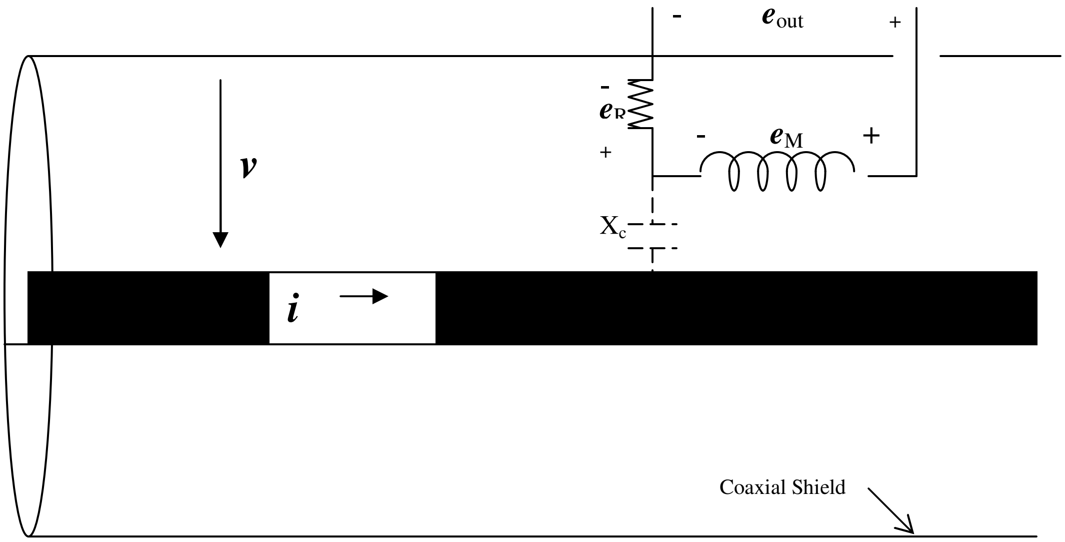

Figure 2: Definitions

The Analysis:

The voltage across the terminating resistor R is a simple voltage divider:

$$e_R = \left[\frac{R}{R + X_c} \right] v$$

We can make a simplifying assumption that $R \langle\langle X_c$ so that

[1] $$e_R = \frac{R}{X_c} v$$

Since $X_c = 1/j\omega C = -j/\omega C$, e_R becomes

[2] $$e_R = -j\omega CRv$$ and

[3] $$e_M = -i \cdot j\omega M$$

For the case where the inductance is rotated 180^0,

[4] $$e_M = i \cdot j\omega M$$

In any event,

[5] $$e_{out} = e_R + e_M$$

Considering the negative (or "reverse") case, and substituting in equations [2] and [4],

[6] $$e_{out} = e_R + e_M = j\omega CRv - i \cdot j\omega M = j\omega(Mi - CRv)$$

When the coaxial line is properly terminated, the output voltage the output voltage of a properly designed directional coupler will be zero in the "reverse" position. An examination of [6] shows that this happens when

$$Mi = CRv$$

Recalling that under conditions of no reflected power that $\frac{v}{i} = Z_0$, the zero reflection point occurs when we set

$$\frac{M}{CR} = Z_0 \quad \text{or when}$$

[7] $$R = \frac{M}{CZ_0}$$

This is a fortunate conclusion since because both M and C are difficult to measure, the balance point can be found simply by adjusting R on a properly terminated line until the reverse output voltage is zero.

Continuing on, and assuming that balance has been achieved, we know from transmission line theory that under conditions of IMPROPERLY terminated transmission lines a reflected (or reverse) wave exists on the line. We know that the voltage at any point along the line is the sum of the voltages created from the forward and the reverse travelling waves. In other words,

[8] $$v = v_F + v_R \quad \text{and the current is}$$

[9] $$i = \frac{v_F}{Z_0} - \frac{v_R}{Z_0}$$

[Incidentally, you might think that the equation would be similar to the voltage case $i = i_F + i_R$ but since current is measured against the wave propagation direction, the negative sign appears in front of the reverse term and the equation becomes $i = i_F - i_R$]

.

Recalling [7] and substituting into [6] for the reverse case

[7] (re-written) $$CR = \frac{M}{Z_0}$$

$$e_{out} = j\omega\left[M\left(\frac{v_F}{Z_0} - \frac{v_R}{Z_0}\right) - \frac{Mv}{Z_0}\right] = \frac{j\omega M}{Z_0}\left[v_F - v_R - v\right]$$

And since [8] $v = v_F + v_R$,

[10] $$e_{out} = \frac{j\omega M}{Z_0}\left[v_F + v_R - v_F + v_R\right] = 2v_R\frac{j\omega M}{Z_0}$$

Considering the forward mode,

[11] $$e_{out} = -2v_F\frac{j\omega M}{Z_0}$$

the minus sign goes away when we consider that the definition of the voltage across M has reversed when it is rotated. So the results become similar:

[12] $$\boxed{e_{out(fwd)} = 2v_F\frac{j\omega M}{Z_0} \quad \text{and} \quad e_{out(rev)} = 2v_R\frac{j\omega M}{Z_0}}$$

What we have now is a useful device that can be used to measure SWR from the formula

[13] $$\text{SWR} = \frac{v_F + v_R}{v_F - v_R} = \frac{e_{out(fwd)} + e_{out(rev)}}{e_{out(fwd)} - e_{out(rev)}}$$

This is a very useful device and is known under the generic names of “SWR METER” or a “Reflectometer”. Typically, a forward voltage is set to a full scale reading with a sensitivity potentiometer and then SWR is read directly off of a meter scale when the loop is reversed. (Rather than physically reversing the loop, the termination resistor can be electrically switched from one end of the loop to the other and this is what is normally done for mechanical simplicity.)

Notice, however, that the sensitivity (magnitude of e_{out}) is a direct function of frequency and that sensitivity increases with increasing frequency. How can we turn this into a less frequency-dependent design? In other words, how can we make this a useful POWER meter over a wide range of frequencies?

Frequency Independence

A quick look at equations [10] and [12] gives us a clue. If we were to place a capacitor across the output, the $j\omega$ terms might cancel out. This turns out to be the case :

Calling the load capacitor C_{out}, the output voltage becomes

[14] $$e_{corrected} = \frac{X_{Cout}}{j\omega L + j\omega M - jX_{Cout} + R} * e_{out} \qquad \text{where } e_{out} = 2v_F \frac{j\omega M}{Z_0} \text{ from eq [7]}$$

and L is the self-inductance of the pickup loop.

An opportunity can be seen here in that if X_{Cout} is chosen so that

[15] $$X_{Cout} = \omega L + \omega M$$

Equation [14] reduces to

[16] $$e_{corrected} = \frac{X_{Cout}}{R} * \; 2v_F \frac{j\omega M}{Z_0} \qquad \text{Recalling from [7] that } \frac{M}{R} = CZ_0$$

[17] $$\boxed{e_{corrected} = \frac{2C}{C_{out}} v_F}$$

Some authors (1) have reduced the assumption in [15] to

$$X_{Cout} << R$$

But this is close to the same assumption since L and M are small w.r.t. R also. In any event, if X_{Cout} can be selected to tune out the inductance contributions of L and M, a wide range of frequencies can be practically achieved with relatively constant output sensitivity.

Note that this forces C_{out} to be a very large value, and since C L and M are already known to be small, this results in significant attenuation of the signal to be measured. Nevertheless, we now have a reasonable and somewhat frequency-independent WATTMETER. (Actually the device is measuring the voltage of the forward and reverse waves independently, but with good directivity, this can be equated to Power.)

Frequency Independence: an alternative method

There is an alternative method used in the past that accomplishes nearly the same result, although there are more compromises involved and is often more difficult to implement. This method increases the self inductance of the pickup winding L_x. A commonly used method would be to encircle the center conductor of the transmission line with a toroid core and then wind multiple turns onto the toroid. This in effect makes a ½ turn "primary" winding and a multiple # of turns on the secondary. This increases the self inductance of the secondary winding L_x (identified as the pickup loop in figure 1 or where e_m is generated in figure 2).

If we eliminate the output load capacitor C_{out} that we added above and replace it with a resistor which we call R_{out} , and recalling from eq. (12) that

$$e_{out} = 2v_F \frac{j\omega M}{Z_0} \qquad \text{(unloaded output)}$$

then, with the output terminated in R_{out} the output voltage becomes

$$e_{out} = 2v_F \frac{j\omega M}{Z_0} * \frac{R_{out}}{(R_{out} + Z_c)} \tag{18}$$

where Z_c is the coupler output impedance. If we let Z_x represent the self inductance of the secondary winding, then we can see that

$$Z_c = Z_m + Z_x + R \qquad \text{(R is the terminating resistance defined in fig 1)}$$

Then [18] becomes

$$e_{out} = 2v_F \frac{j\omega M}{Z_0} * \frac{R_{out}}{(R_{out} + Z_m + Z_x + R)} \tag{19}$$

Here, we see the frequency independence opportunity emerge from the equation: since $Z_m + Z_x = j\omega(M + L_x)$.

If we choose our $j\omega(L_x + M) >> R + R_{out}$ we see that the undesired terms $j\omega$ nearly drop out of [19].

The final equation then would then become

$$e_{out} = 2v_F \frac{M}{Z_0} * \frac{R_{out}}{(M + L_x)} \tag{20}$$

and finally results in

[21] $$e_{out} = 2v_F \frac{M}{Z_0} * \frac{R_{out}}{L_x} = \frac{2v_f M R_{out}}{Z_o L_x} \qquad \text{(since } L_x{>>}M\text{)}$$

Previously in this section we made the assumption that $j\omega(L_x + M) >> R + R_{out}$
Now let's check to make sure that our assumption results in a reasonable value for L_x :

Assume a reasonable number for $R + R_{out} \sim 100$ ohms. To be valid within 1% then, our assumption for $j\omega(L_x + M)$ must be >> 10000 ohms
If we choose our lowest frequency of interest to be 3.5MHz, and since $\omega = 2\pi f$ then

$$(L_x + M) >> \frac{10000}{2\pi(3.5x10^6)} = 454\,\mu Hy$$

This value of L_x alone could easily be realized with a ferrite core of $A_L = 1.9$ and wound with about 15 or 16 turns. [$L = N^2 A_L$]

This is a good approach with the assumptions getting better with increasing frequency, but the secondary coil must encircle the center conductor of coaxial line making it more and more difficult to maintain a smooth insertion impedance as frequency increases.

Comments on Directivity

Directivity is one measure of the quality of an RF coupler. In a coaxial system where the coax is terminated in its characteristic impedance, the directivity is the magnitude of the forward wave divided by the magnitude of the reflected wave. This will be ∞ and is often referred to as RETURN LOSS. Practical couplers will have a directivity of >-30 or -40db. This will give good readings of forward power since the reverse power interaction is then insignificant. With good directivity, and since the reflected wave is not interacting, the measurement of the forward VOLTAGE is directly proportional to forward POWER.

Note also, that since directivity is a ratio of equations [10] and [12], so directivity is (almost … where the assumptions are still correct) FREQUENCY INDEPENDENT.

The next step is to detect the voltages and convert them to POWER readings.

Return to home page at http://www.k9so.net for the next installment in the investigation of directional couplers: RF Detectors, Square Law and Peak

References:

(1)
HF Antenna Collection, edited by Erwin David, G4LQI
2nd edition
Published by RSGB in 1994. Originally published June, 1968 Radcom
By M M Bibby, G3NJY
The reader is cautioned that this edition has several typographical errors in some of the formulae presented.