RF DETECTORS: square law, peak, and transitional modes

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This paper is an analysis of the function of the diode used in the detection part of directional RF detectors. The ultimate goal is to understand the scale on the 30uA Bird wattmeter. It is the assertion of this paper that the diode can act as a square law detector and not as a “rectifier” for low input signals. For small signals, the output of the Low Pass Filter (LPF) in figure 1 is a DC voltage directly proportional to the POWER dissipated in $\mathbf{R_S}$. (The LPF can simply be a capacitor placed across $\mathbf{R_L}$)

Consider the following circuit and the typical diode curve in the forward bias region. This looks like a half-wave rectifier circuit, but it’s not always so.

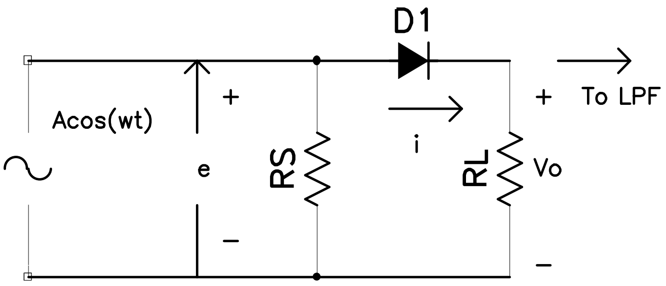


Figure 1

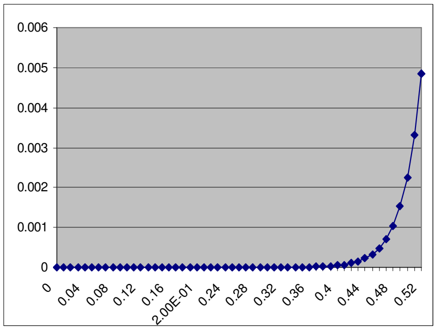
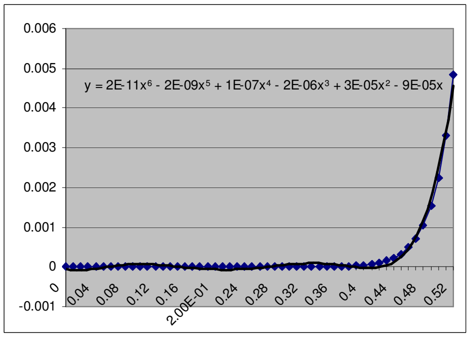


Figure 2: Typical Diode V-I curve

The following assumptions are made for the sake of simplicity of analysis:

1. The diode need not have any forward bias to function, but for now, assume that a very small signal is operating in the knee of the curve by adding a small bias current.
2. $A\cos(\omega t) << V_{diode}$ **<u>Note that under these conditions, *the signal is not rectified*</u>**
3. The V-I curve of the series combination of D_1 and R_L can be represented by a power series of the form:

$$\mathbf{i_d = ae + be^2 + ce^3 + \ldots}$$



4. $R_{diode} + R_L >> R_S$

Figure 3: Polynomial representation of diode curve

ANALYSIS of the First Assumption:

(small signal)

Since $e = A\cos(\omega t)$, i_d may be represented by the polynomial

[1] $\mathbf{i_d = a\,A\cos(\omega t) + bA^2\cos^2(\omega t) + cA^3\cos^3(\omega t) + \ldots}$

(the R_s term is absorbed in the constants a, b, c, etc.)

In trig, there are identities that are useful. Here are two of them:

[2] $\mathbf{\cos^2(\omega t) = (1/2)[1+\cos(2\omega t)]}$ **and**

[3] $\mathbf{\cos^3(\omega t) = (1/4)[\cos(3\omega t) + 3\cos(\omega t)]}$

Substituting [2] and [3] (and the continuing infinite list of identities) into equation [1] gives us the following result:

[4] $\mathbf{i_d = aA\cos(\omega t) + (b/2)A^2[1 + \cos(2\omega t)] + (c/4)A^3[\cos(3\omega t) + 3\cos(\omega t)] + \ldots.}$

[5] $\mathbf{= (b/2)A^2 + [aA + (3/4)\,cA^3]\cos(\omega t) + (b/2)A^2(2\omega t) + (c/4)A^3\cos(3\omega t) + \ldots}$

Again, simplifying the constants gives an equation of the form

[6] $\mathbf{\gamma_1 A^2 + \gamma_2\cos(\omega t) + \gamma_3\cos(2\omega t) + \gamma_4\cos(3\omega t) + \ldots}$ (where γ_n are constants)

Note that a DC term $\gamma_1 A^2$ has appeared.
Since the voltage across R_L (v_o) is what is of interest, and $v_o = i_d R_L$

[7] $\mathbf{v_o = R_L\gamma_1 A^2 + R_L\gamma_2\cos(\omega t) + R_L\gamma_3\cos(2\omega t) + R_L\gamma_4\cos(3\omega t) + \ldots}$

This represents a DC term + all the harmonics of $\cos(\omega t)$. By passing $\mathbf{v_o}$ through a low pass filter (a simple output capacitor) we can output only the DC output voltage term

[8] $\mathbf{V_O = R_L\gamma_1 A^2}$

This is a DC voltage proportional to the SQUARE of the amplitude of the input RF signal!

Remembering that **A** is the amplitude of that RF signal, and that Power P = (V^2/R), the peak power dissipated in R_S is

[9] $$\mathbf{P=A^2/\,R_S}$$

Solving equation [8] for $\mathbf{A^2}$, equation [9] becomes

[10] $$\mathbf{P= V_O / [R_L\, R_S\, \gamma_1]}$$

This is of the form:

[11] $$\mathbf{P = \alpha V_O}$$

where α is a constant and $\mathbf{V_O}$ is the DC term at the output of the LPF

*** In other words, V_o is directly proportional to the ***power*** dissipated by $\mathbf{R_S}$. ***

A different assumption:

(large signal)

The above analysis is based on the assumption that $A\cos(\omega t) << V_{diode}$ and that the signal is not being rectified by the diode. Let's now look at this detector using the opposite assumption. That is to say that the diode acts as a rectifier and that

[12] $$\mathbf{A \cos(\omega t) >> V_{diode}}$$

With the addition of the LPF (capacitor) at the output, and again making the assumption that there is no detector circuit loading ($R_{diode} + R_L >> R_S$), then

[13] $$\mathbf{V_O = A}$$

where A is the amplitude of the input sinusoid. The circuit becomes a simple voltage peak detector (again assuming that $\mathbf{V_{diode}}$ is small) and the power dissipated in $\mathbf{R_S}$ is related to the square of $\mathbf{V_O}$ rather than being a linear function of $\mathbf{V_O}$ as concluded previously. That is to say that

[14] $\mathbf{P = \alpha (V_O)^2}$ for the peak (large signal) detector and

[11] $\mathbf{P = \alpha V_O}$ for the square law (small signal) detector.

So, for very small signal levels, the voltage varies LINEARLY with the power input (as a square law detector) but at large signal levels, the SQUARE ROOT of the voltage varies with the input power (as a peak detector).

Which way is best? Well, the small signal model must be amplified in order to be properly displayed. It also requires a small DC bias for best “linearity”. Both require a power supply to be connected to the circuit. However the large signal model can potentially introduce distortion in the signal being measured and will likely generate harmonics and/or intermodulation products. It will likely also suffer temperature-related errors due to changes in the peak voltage (the indicated peak will be the actual peak voltage minus the diode forward voltage).

A look at a self-powered portable wattmeter

A typical schematic might look like this:

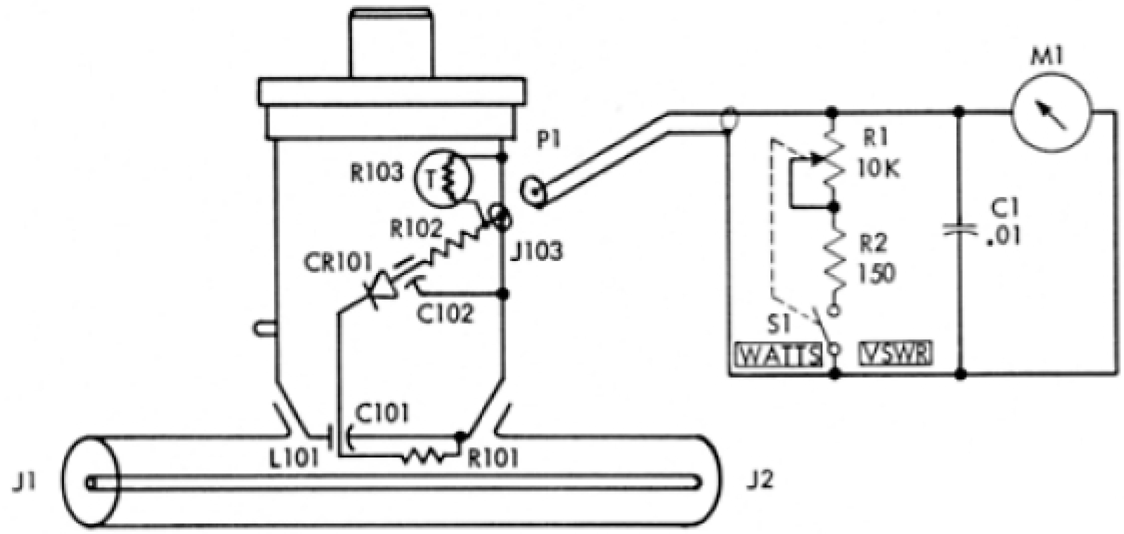


Figure 4: a variation of the Bird wattmeter theme

With only minor variations, this is used by many manufacturers of portable wattmeters. About the only thing that changes from manufacturer to manufacturer is the direction of the diode. That will simply change the polarity of the metering connection.

While schematically different from figure 1, the detection analysis is similar with CR101 acting as the detector and C102 as the LPF. R102 is an attenuation and calibration resistor. What we’d like to understand is if this is operating as a square law detector or as a peak detector.

Obviously, the first clues are the lack of any DC bias method for the diode and the thermistor at the output. Both are characteristic of the peak detector mode of operation.

So, for this peak detector mode of operation, we would expect a current vs. “wattmeter reading” as shown in figure 5 below:

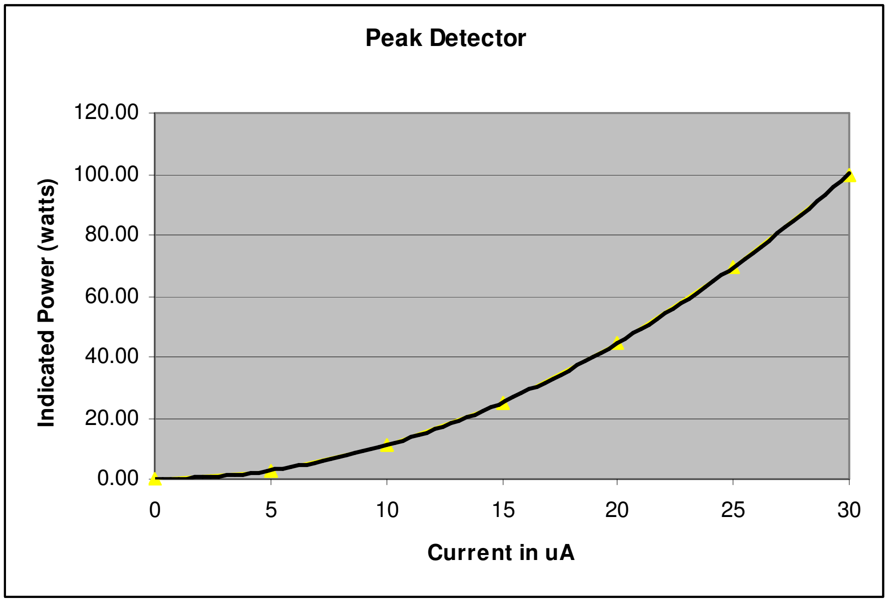


Figure 5: expected scale of a peak detector wattmeter

To generate this graph, the current scale was normalized to a full scale of 10 and then the value was squared since from equation [14], we know that $\mathbf{P = \alpha (V_O)^2}$

But it would seem obvious that as the power gets reduced, at some point the diode will stop rectifying (simply because the signal is so small) and start acting more like a square law detector. The expected output from a square law detector is shown in figure 6:

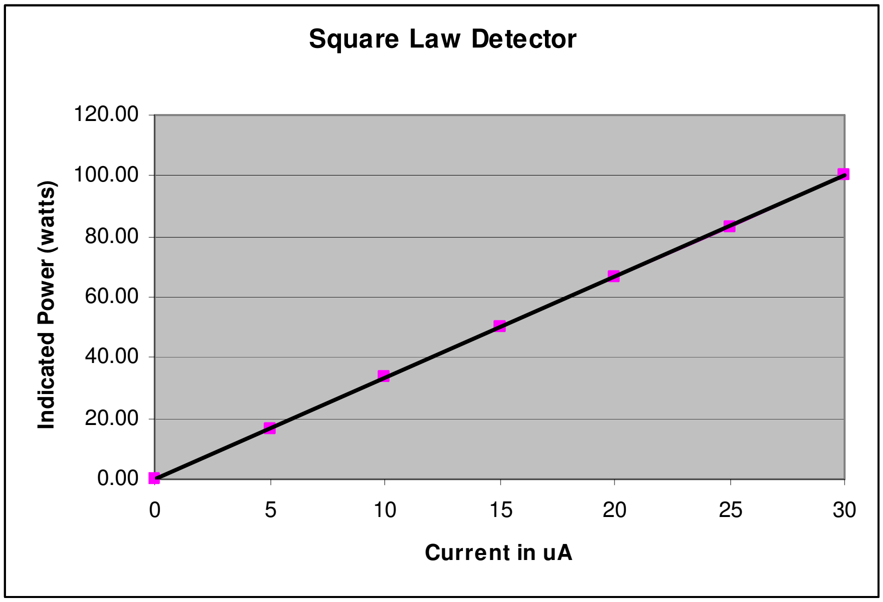


Figure 6: expected scale of a “square law” detector: a straight line

This graph was generated the same way as figure 3, but here equation [11] was used:

$$\mathbf{P} = \alpha \mathbf{V_O}$$

A clue to what modes these wattmeters operate in can be uncovered by actually measuring the indicated watts at the same current inputs. This data was taken from a 30uA Bird meter:

Current	Indicated Power (watts)
0	0.00
5	9.50
10	22.20
15	38.50
20	56.10
25	80.00
30	100.00

This was plotted alongside the data of figures 5 and 6:

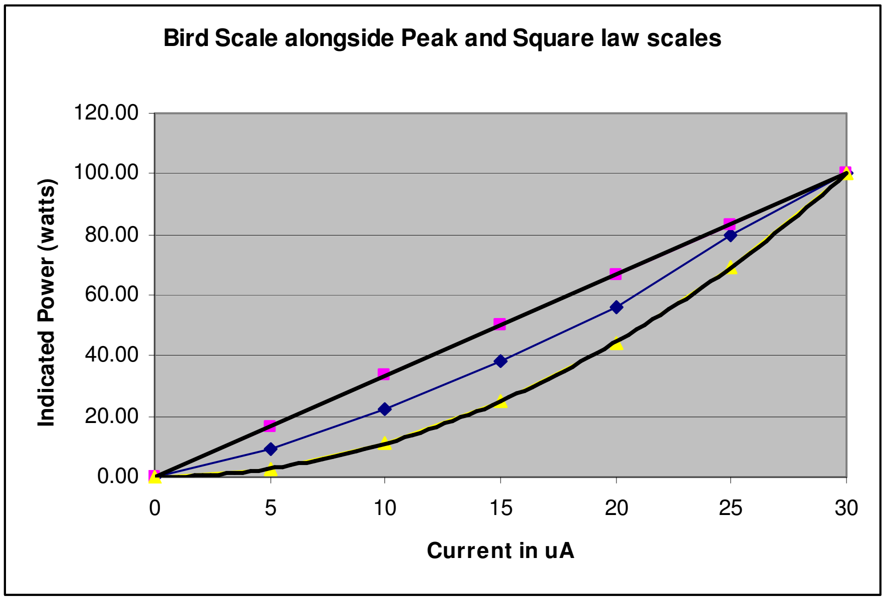


Figure 7

This clearly shows that the meter is operating in between the two detector modes. Call this the "transition" mode.

Another way of looking at this is to calculate the power to which the x-axis (normalized current) data must be raised to get to the indicated level. Once again, this was normalized and the result is plotted below:

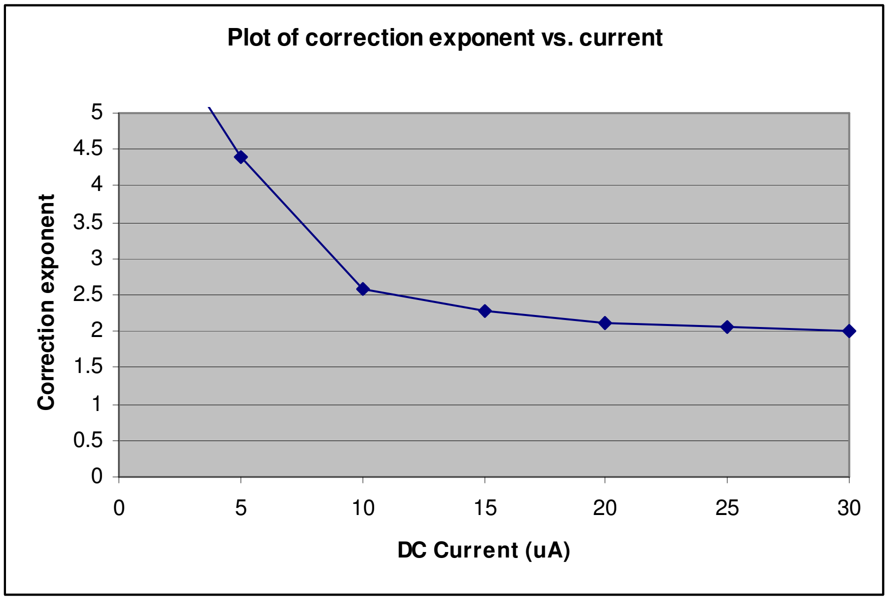


Figure 8

Figure 8 clearly shows that as the signal increases, the detector in the Bird wattmeter acts more and more like a peak detector. This is evidenced by the fact that the exponent asymptotically approaches the expected value of 2 for the peak detector mode.

This is useful information if the wattmeter scales are to be increased beyond the 30uA limit of the typical “Bird”. We can assume that the extended scale would follow the peak detector mode curve:

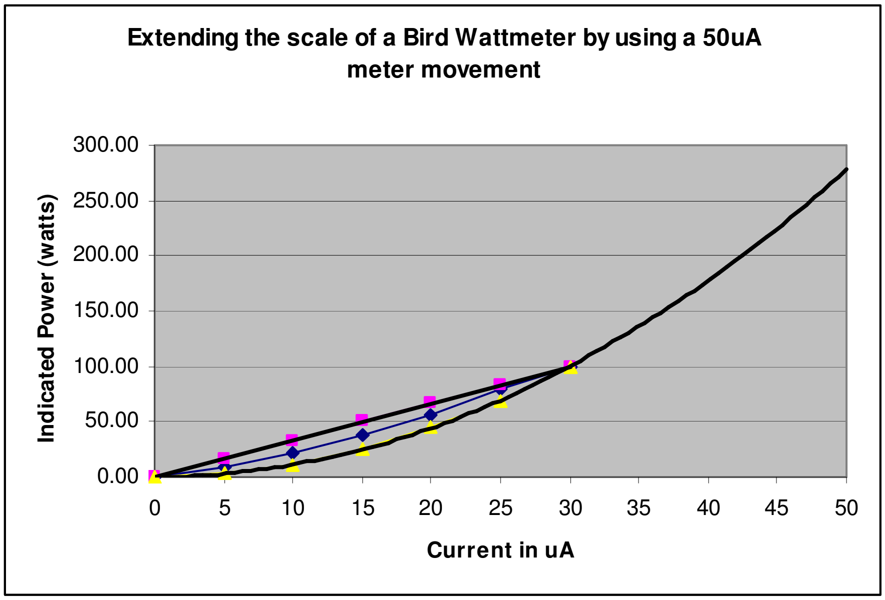


Figure 9: an extended scale will follow the curve of a peak detector beyond 30uA

This is useful information for the replacement meter project and how to adjust the meter scale.